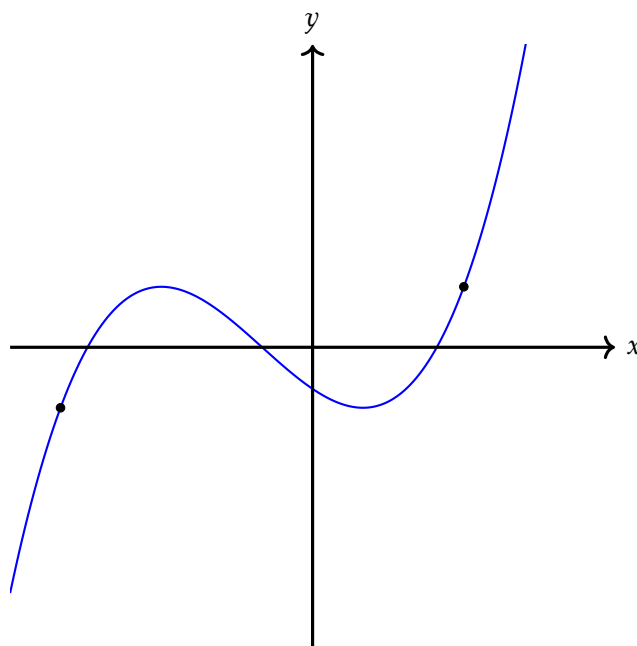
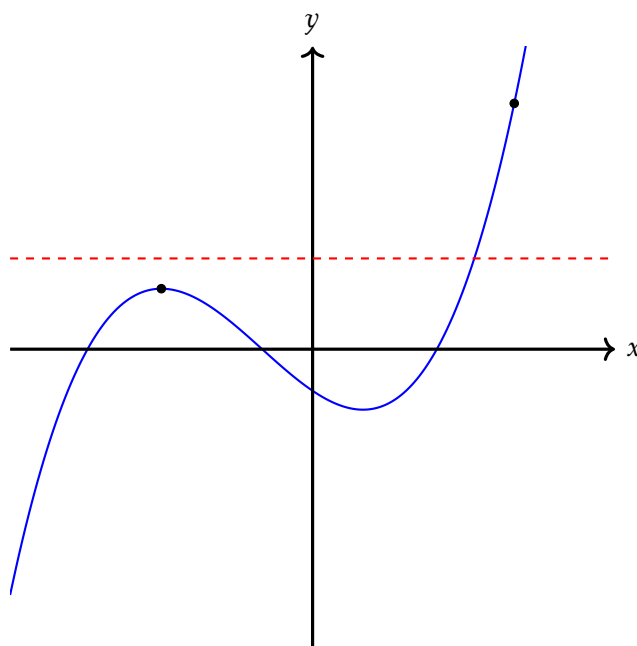


## Change of Sign



**Fact (Sign-change rule)** — If  $f$  is a continuous function and if  $f(a) < 0$  and  $f(b) > 0$  then there is a some value  $c$  between  $a$  and  $b$  such that  $f(c) = 0$

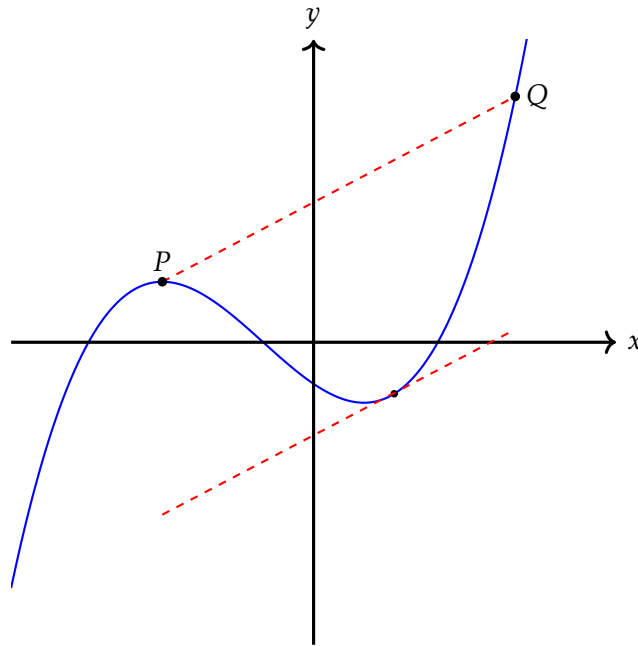
## Intermediate Value Theorem



**Fact (Intermediate Value Theorem)** — If  $f$  is a continuous function then for all values  $\min(f(a), f(b)) \leq y \leq \max(f(a), f(b))$  there is some value  $c$  such that  $f(c) = y$

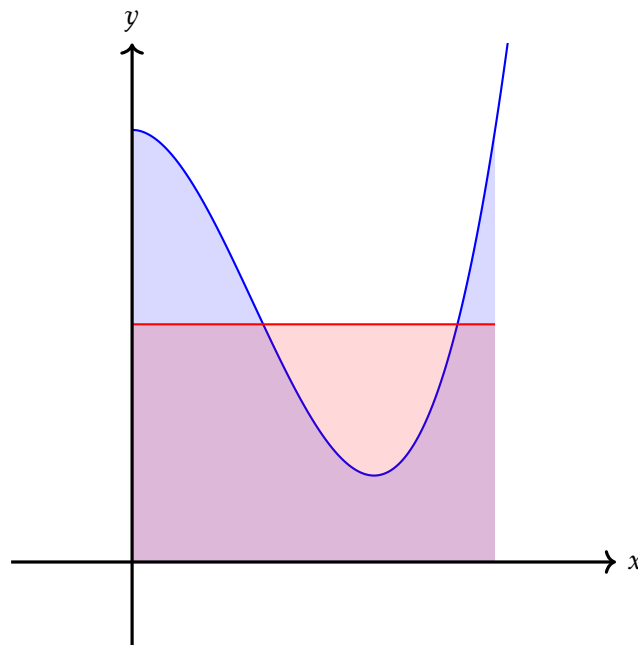
**Fact** — The sign-change rule is a special case of the intermediate value theorem

## Mean Values



**Fact** — If  $f$  is a function with *continuous* derivative, and points  $P = (a, f(a))$  and  $Q = (b, f(b))$  then there is a point  $c \in (a, b)$  with tangent parallel to the line joining the points, ie

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



**Definition.** The **mean value** of a function on  $(a, b)$  is  $\frac{1}{b-a} \int_a^b f(x) dx$

**Example**

Find the mean value of  $f(x) = x^3 + 2x + 1$  on  $(1, 3)$

**Example**

The speed,  $v$  metres per second, of a car  $t$  seconds after the driver presses the accelerator can be modelled by the equation  $v = 25(1 - e^{-0.3t})$  for  $t \geq 0$ . Determine, according to the model, the mean speed of the car over the first 10 seconds after the accelerator is pressed. Give your answer correct to 2 decimal places.